# Quantum statistical properties of the Jaynes-Cummings model in the presence of a homogeneous gravitational field

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**Abstract**

The temporal evolution of quantum statistical properties of an interacting atom-field system in the presence of a homogeneous gravitational field is investigated within the framework of the Jaynes-Cummings model. Taking into account both the atomic motion and gravitational field a full quantum treatment of the internal and external dynamics of the atom is presented based on an alternative su(2) dynamical algebraic structure. By solving analytically the Schrödinger equation in the interaction picture, the evolving state of the system is found by which the influence of the gravitational field on the dynamical behavior of the atom-field system is explored. Assuming that initially the field is prepared in a coherent state and the two-level atom is in a coherent superposition of the excited and ground states, the influence of gravity on the atomic dipole moment, collapses and revivals of the atomic motion, atomic momentum diffusion, photon counting statistics and quadrature squeezing of the radiation field is studied.

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## 1 Introduction

The interaction between a two-level atom and a single quantized mode of the electromagnetic field in a lossless cavity within the rotating wave approximation (RWA)can be described by the Jaynes-Cummings model (JCM) [1]. Despite being simple enough to be analytically soluble in the RWA, this model has been a long-lasting source of insight into the nuances of the interaction between light and matter. The JCM has been applied to investigate many quantum effects such as the quantum collapses and revivals of atomic inversion [2], squeezing of the radiation field [3] ,atomic dipole squeezing [4], vacuum Rabi oscillation [5] and the dynamical entangling and disentangling of the atom-field system in the course of time [6-9]. Investigations of the dynamical behavior of the JCM are also extremely important due to its experimental realizations in high-Q microwave cavities [10], in optical resonators [11], in laser-cooled trapped ions [12] and in quantum nondemolition measurements [13]. Stimulated by the success of the JCM, more and more people have paid special attention to extending and generalizing the model in order to explore new quantum effects. Discussions related to several interesting generalizations of this model are now available in the literature [14] and the model is still promising in many applications, particularly in the tast developing research area of quantum information [15].

A very significant and noteworthy generalization of the JCM is to include the effect of atomic motion so that the spatial mode structure could be incorporated into this model. In the standard JCM, the interaction between a constant electric field and a stationary (motionless) two-level atom is considered. With the development in the technologies of laser cooling and atom trapping the interaction between a moving atom and the field has attracted much attention [16-25]. In particular, it has been shown that the atomic motion can bring about the nonlinear transient effects similar to self-induced transparency (SIT) and adiabatic following (AF) [26], the possibility of realizing an optical switching [25], change the creating time of Schrödinger cat states [22] and exhibit long time entropy squeezing effect [24].

On the other hand, experimentally, atomic beams with very low velocities are generated in laser cooling and atomic interferometry [27]. It is obvious that for atoms moving with a velocity of a few millimeters or centimeters per second for a time period of several milliseconds or more, the influence of Earth's acceleration becomes important and cannot be neglected [28]. For this reason it is of interest to study the temporal evolution of a moving atom simultaneously exposed to the gravitational field and a singel-mode travel-

ing wave field. Since any quantum optical experiment in the laboratory is actually made in a non-inertial frame it is important to estimate the influence of Earth's acceleration on the outcome of the experiment. Recently, a semiclassical description of a two-level atom interacting with a running laser wave in a gravitational field has been studied [29,30]. In Ref.[31] a complementary scheme based on an su(2) dynamical algebraic structure to investigate the influence of the gravity on the QND measurement of atomic momentum in the dispersive JCM has been studied.

In this paper we adopt a dynamical algebraic approach to investigate the temporal evolution of quantum statistical properties of the JCM in the presence of a homogeneous gravitational field. In the Jaynes-Cummings model, when the atomic motion is in a propagating light wave, we consider a two-level atom interacting with the quantized cavity-field in the presence of a homogeneous gravitational field. By solving analytically the Schrödinger equation in the interaction picture, the evolving state of the system is found by which the influence of the gravitational field on the dynamical behavior of the atom-field system is explored. In section 2, we present a full quantum treatment of the internal and external dynamics of the atom with an alternative su(2) dynamical algebraic structure within the system. Based on this su(2) structure and in the interaction picture, we obtain an effective Hamiltonian describing the atom-field interaction in the presence of a gravitational field. In section 3 we investigate the dynamical evolution of the system and show that how the gravitational field may affect the dynamical properties of the JCM. In section 4 we study the influence of gravitational field on both the cavity-field and the atomic properties. Considering the field to be initially in a coherent state and the two-level atom in a coherent superposition of the ground and excited states, we investigate the temporal evolution of the atomic dipole moment, atomic inversion, atomic momentum diffusion, probability distribution of the cavity-field, photon counting statistics and quadrature squeezing of the radiation field. Finally, we summarize our conclusions in section 5.

## 2 Jaynes-Cummings Model in the presence of Gravitational Field

The system we consider here is a moving two-level atom of mass M exposed simultaneously to a single-mode travelling wave field and a homogeneous gravitational field. Therefore, the Hamiltonian of the atom-field system in the presence of gravitational field with the atomic motion along the position

vector $\hat{\vec{x}}$ and in the rotating wave approximation is given by

$$\begin{aligned}\hat{H} = \quad & \frac{\hat{p}^2}{2M} - M\vec{g}.\hat{\vec{x}} + \hbar\omega_c(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + \frac{1}{2}\hbar\omega_{eg}\hat{\sigma}_z + \\ & \hbar\lambda[\exp(-i\vec{q}.\hat{\vec{x}})\hat{a}^\dagger\hat{\sigma}_- + \exp(i\vec{q}.\hat{\vec{x}})\hat{\sigma}_+\hat{a}],\end{aligned} \tag{1}$$

where $\hat{a}$ and $\hat{a}^\dagger$ denote, respectively, the annihilation and creation operators of a single-mode traveling wave with frequency $\omega_c$, $\vec{q}$ is the wave vector of the running wave and $\hat{\sigma}_\pm$ denote the raising and lowering operators of the two-level atom with electronic levels $|e\rangle, |g\rangle$ and Bohr transition frequency $\omega_{eg}$. The atom-field coupling is given by the parameter $\lambda$ and $\hat{\vec{p}}$, $\hat{\vec{x}}$ denote, respectively, the momentum and position operators of the atomic center of mass motion and $g$ is Earth's gravitational acceleration. It has been shown [31] that based on su(2) algebraic structure, as the dynamical symmetry group of the model, the Hamiltonian (1) can be transformed to the following effective Hamiltonian

$$\hat{\tilde{H}} = \frac{\hat{p}^2}{2M} - \hbar\hat{\triangle}(\hat{\vec{p}},\vec{g})\hat{S}_0 + \frac{1}{2}Mg^2t^2 + \hat{\vec{p}}.\vec{g}t + \hbar(\kappa\sqrt{\hat{K}}\hat{S}_- + \kappa^*\sqrt{\hat{K}}\hat{S}_+), \tag{2}$$

where $\hat{\kappa}(t)$ is an effective coupling coefficient

$$\hat{\kappa}(t) = \lambda\exp(\frac{it}{2}(\hat{\triangle}(\hat{\vec{p}},\vec{g}) + \frac{\hbar q^2}{M})), \tag{3}$$

and the operators

$$\hat{S}_0 = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|), \hat{S_+} = \hat{a}|e\rangle\langle g|\frac{1}{\sqrt{\hat{K}}}, \hat{S_-} = \frac{1}{\sqrt{\hat{K}}}|g\rangle\langle e|\hat{a}^\dagger, \tag{4}$$

with the following commutation relations

$$[\hat{S}_0, \hat{S_\pm}] = \pm\hat{S_\pm}, [\hat{S_-}, \hat{S_+}] = -2\hat{S}_0, \tag{5}$$

are the generators of the su(2) algebra and the operator

$$\hat{\triangle}(\hat{\vec{p}},\vec{g}) = \omega_c - (\omega_{eg} + \frac{\vec{q}.\hat{\vec{p}}}{M} + \vec{q}.\vec{g}t + \frac{\hbar q^2}{2M}), \tag{6}$$

has been introduced as the Doppler shift detuning at time $t$ [31]. The Hamiltonian (2) has the form of the Hamiltonian of the JCM, the only modification being the dependence of the detuning on the conjugate momentum and the

gravitational field. In the interaction picture the transformed Hamiltonian (2) takes the following form

$$\hat{\tilde{H}}_{int} = \exp(\frac{-i\hat{\tilde{H}}_0 t}{\hbar})\hat{\tilde{H}}_I \exp(\frac{i\hat{\tilde{H}}_0 t}{\hbar}), \tag{7}$$

where

$$\hat{\tilde{H}}_0 = -\hbar\hat{\triangle}(\hat{\vec{p}},\vec{g})\hat{S}_0 + \hat{H}(\hat{\vec{p}},\vec{g}), \tag{8}$$

and

$$\hat{\tilde{H}}_I = \hbar(\kappa\sqrt{\hat{K}}\hat{S}_- + \kappa^*\sqrt{\hat{K}}\hat{S}_+), \tag{9}$$

with

$$\hat{H}(\hat{\vec{p}},\vec{g}) = \frac{\hat{p}^2}{2M} + \hat{\vec{p}}.\vec{g}t + \frac{1}{2}Mg^2t^2. \tag{10}$$

Therefore we obtain

$$\hat{\tilde{H}}_{int} = \hbar(\hat{\kappa}(t)\sqrt{\hat{K}}\hat{S}_- \exp(-it\hat{\triangle}(\hat{\vec{p}},\vec{g})) + \hat{\kappa}^*(t)\sqrt{\hat{K}}\hat{S}_+ \exp(it\hat{\triangle}(\hat{\vec{p}},\vec{g}))). \tag{11}$$

Finally by using Eq.(3) we arrive at

$$\hat{\tilde{H}}_{int} = \hbar\lambda(\sqrt{\hat{K}}\hat{S}_- \exp(-it\hat{\triangle}_1(\hat{\vec{p}},\vec{g},t)) + \sqrt{\hat{K}}\hat{S}_+ \exp(it\hat{\triangle}_1(\hat{\vec{p}},\vec{g},t))). \tag{12}$$

where

$$\hat{\triangle}_1(\hat{\vec{p}},\vec{g},t) = \frac{1}{2}(\omega_c - (\omega_{eg} + \frac{\vec{q}.\hat{\vec{p}}}{M} + \vec{q}.\vec{g}t + 3\frac{\hbar q^2}{2M})). \tag{13}$$

is the detuning of the atom-field interaction which depends on both the atomic momentum and the gravitational field.

## 3 Dynamical Evolution

In section 2, we obtained an effective Hamiltonian for the atom-field system in the presence of a homogeneous gravitational field in the interaction picture. In this section, we investigate dynamical evolution of the system. We will show how the gravitational field may affect the quantum dynamics of JCM. For this purpose, we solve the Schrödinger equation

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = \hat{\tilde{H}}_{int}|\psi\rangle, \tag{14}$$

for the state vector $|\psi(t)\rangle$ with the Hamiltonian (12). Indeed, the two-level atom with momentum $|\vec{p}\rangle$ in the excited state $|e\rangle$ get annihilated and

creates a field excitation. Therefore, the Hamiltonian $\hat{\tilde{H}}_{int}$ transforms the state $|e\rangle \otimes |n\rangle \otimes |\vec{p}\rangle \equiv |e,n\rangle \otimes |\vec{p}\rangle$, where $|n\rangle$ denotes the nth Fock state of the field, into

$$\hat{\tilde{H}}_{int}|e,n\rangle \otimes |\vec{p}\rangle = \hbar\lambda\sqrt{n+1}\exp(-it\hat{\triangle}_1(\vec{p},\vec{g},t))|g,n+1\rangle \otimes |\vec{p}\rangle, \tag{15}$$

in which we have used the relations

$$\sqrt{\hat{K}}\hat{S}_-|e,n\rangle = \sqrt{n+1}|g,n+1\rangle, \hat{\vec{p}}|\vec{p}\rangle = \vec{p}|\vec{p}\rangle. \tag{16}$$

Similarly, atom with momentum $|\vec{p}\rangle$ in the ground state $|g\rangle$ get excited at the expense of annihilation a field excitation. Hence, the Hamiltonian transforms the state $|g\rangle \otimes |n+1\rangle \otimes |\vec{p}\rangle \equiv |g,n+1\rangle \otimes |\vec{p}\rangle$ into

$$\hat{\tilde{H}}_{int}|g,n+1\rangle \otimes |\vec{p}\rangle = \hbar\lambda\sqrt{n+1}\exp(it\hat{\triangle}_1(\vec{p},\vec{g},t))|e,n\rangle \otimes |\vec{p}\rangle. \tag{17}$$

Since the Hamiltonian couples only the states $|g,n+1\rangle \otimes |\vec{p}\rangle$ and $|e,n\rangle \otimes |\vec{p}\rangle$ we introduce the state vector

$$\begin{aligned} |\psi(t)\rangle = & \int d^3p \sum_{n=0} (\psi_{e,n}(\vec{p},\vec{g},t)|e,n\rangle \otimes |\vec{p}\rangle + \psi_{g,n+1}(\vec{p},\vec{g},t)|g,n+1\rangle \otimes |\vec{p}\rangle) \\ + & \int d^3p\psi_{g,0}(\vec{p},t)|g,0\rangle \otimes |\vec{p}\rangle. \end{aligned} \tag{18}$$

The state $|g,0\rangle$ which corresponds to $n=-1$ in Eq.(17) plays a special role. According to Eq.(17) we find $\hat{\tilde{H}}_{int}|g,0\rangle = 0$ which means, the vacuum cannot excite an atom initially in the ground state and therefore, the state $|g,0\rangle$ decouples from the rest of the states. Now we find the equations of motion for the time-dependent probability amplitudes $\psi_{e,n}(\vec{p},\vec{g},t) = \psi_1$, $\psi_{g,n+1}(\vec{p},\vec{g},t) = \psi_2$ by substituting (18) into (14) and making use of Eqs.(15) and (17)

$$\dot{\psi}_1 = -i\lambda\sqrt{n+1}\exp(i\triangle_1(\vec{p},\vec{g},t)t)\psi_2, \tag{19}$$

and

$$\dot{\psi}_2 = -i\lambda\sqrt{n+1}\exp(-i\triangle_1(\vec{p},\vec{g},t)t)\psi_1. \tag{20}$$

At time $t=0$ the atom is uncorrelated with the field and the state vector of the system can be written as a direct product

$$\begin{aligned} |\psi(t=0)\rangle = & |\psi_{c.m}(0)\rangle \otimes |\psi_{atom}(0)\rangle \otimes |\psi_{field}(0)\rangle \\ = & (\int d^3p\phi(\vec{p})|\vec{p}\rangle) \otimes (c_e|e\rangle + c_g|g\rangle) \otimes (\sum_{n=0} w_n|n\rangle), \end{aligned} \tag{21}$$

where we have assumed that initially the field is in a coherent superposition of Fock states, the atom is in a coherent superposition of its excited and ground states, and the wave vector for the center-of-mass degree of freedom is $|\psi_{c.m}(0)\rangle = \int d^3p\phi(\vec{p})|\vec{p}\rangle$. In notation (17) the initial state (21) reads

$$\begin{aligned}|\psi(t=0)\rangle = & \int d^3p\sum_{n=0}(w_n c_e\phi(\vec{p})|e,n\rangle\otimes|\vec{p}\rangle + w_{n+1}c_g\phi(\vec{p})|g,n+1\rangle\otimes|\vec{p}\rangle) \\ + & \int d^3p w_0\phi(\vec{p})c_g|g,0\rangle\otimes|\vec{p}\rangle. \end{aligned}\tag{22}$$

When we compare (22) with (18) we find the initial conditions

$$\psi_1(t=0) = w_n c_e\phi(\vec{p}), \psi_2(t=0) = w_{n+1}c_g\phi(\vec{p}), \psi_{g,0}(t=0) = w_0 c_g\phi(\vec{p}). \tag{23}$$

We can solve two coupled first order differential equations (19) and (20) in a straightforward way. We have

$$\frac{\partial^2\psi_1}{\partial t^2} + 2i\vec{q}.\vec{g}(t - \frac{\triangle_0}{2\vec{q}.\vec{g}})\frac{\partial\psi_1}{\partial t} + \lambda^2(n+1)\psi_1 = 0, \tag{24}$$

and

$$\frac{\partial^2\psi_2}{\partial t^2} - 2i\vec{q}.\vec{g}(t - \frac{\triangle_0}{2\vec{q}.\vec{g}})\frac{\partial\psi_2}{\partial t} + \lambda^2(n+1)\psi_2 = 0, \tag{25}$$

where

$$\triangle_0(\vec{p}) = \frac{1}{2}[\omega_c - (\omega_{eg} + \frac{\vec{q}.\vec{p}}{M} + 3\frac{\hbar q^2}{2M})]. \tag{26}$$

is time-independent. Now, we solve analytically these equations and we obtain

$$\psi_1(t) = \exp(i\triangle_1 t)(C(1)H(A_n, B_t) + C(2)_1F_1(-A_n, \frac{1}{2}; B_t^2)), \tag{27}$$

and

$$\psi_2(t) = C(1)H(A_n+1, B_t) + C(2)_1F_1(-\frac{1}{2}(A_n+1), \frac{1}{2}; B_t^2), \tag{28}$$

where $C(1) = \frac{C_1}{C}$,$C(2) = \frac{C_2}{C}$ with

$$C_1 = \psi_1(0)_1F_1(-\frac{1}{2}(A_n+1), \frac{1}{2}; (-D)^2) - \psi_2(0)_1F_1(-A_n, \frac{1}{2}; (-D)^2), \tag{29}$$

and

$$C_2 = \psi_1(0)H(A_n+1, -D) - \psi_2(0)H(A_n, -D), \tag{30}$$

so that

$$C = H(A_n,-D)_1F_1(-\frac{1}{2}(A_n+1),\frac{1}{2};(-D)^2)-H(A_n+1,-D)_1F_1(-A_n,\frac{1}{2};(-D)^2), \tag{31}$$

and we have $A_n = -(2+i\beta)$, $\beta = \frac{\Omega_n(\vec{p},\vec{g})-\Delta_0^2}{2\vec{q}.\vec{g}}$, $B_t = (\gamma t-\eta)(1+i)$, $\gamma = \frac{\sqrt{2}}{2}\vec{q}.\vec{g}$, $\eta = \frac{\sqrt{2}\Delta_0}{4\sqrt{\vec{q}.\vec{g}}}$, $D = \eta(1+i)$. We define $\Omega_n(\vec{p},\vec{g}) = \sqrt{\Omega_n(\vec{p},0)^2+2i\vec{q}.\vec{g}}$ with $\Omega_n(\vec{p},0)^2 = \lambda^2(n+1)+\triangle_0^2$ as the gravity-dependent Rabi frequency and $H(A_n,B_t)$, ${}_1F_1(-A_n,\frac{1}{2};B_t^2)$ as the Hermite and the hypergeometric functions, respectively.

# 4 Dynamical Properties of The Model

In this section, we study the influence of the gravitational field on the quantum statistical properties of the atom and the quantized radiation field.

### 4a. Atomic Dipole Moment

When a two-level atom interacts with the cavity-field, a dipole moment is induced between the two atomic levels. This induced dipole moment is given by the expectation value of the dipole moment operator

$$P(t) = \langle\psi(t)|e\hat{\vec{x}}|\psi(t)\rangle. \tag{32}$$

Therefore, from (18) we obtain

$$P(t) = \int d^3p\sum_{n=0}^{\infty}[\psi_1^*\psi_2\wp_{eg}+\psi_2^*\psi_1\wp_{eg}^*], \tag{33}$$

where

$$\wp_{eg} = \wp_{ge}^* = e\langle e|\hat{\vec{x}}|g\rangle = |\wp_{eg}|\exp(i\varphi), \tag{34}$$

is the dipole matrix element and $\varphi$ is its phase. We assume at $t=0$, atom is in a coherent superposition of the excited state and the ground state $c_g(0) = \frac{1}{\sqrt{2}}$, $c_e(0) = \frac{1}{\sqrt{2}}$, $\varphi = 0$. We now consider gravitational influence on the dipole moment evolution when in $t=0$, the cavity-field is initially prepared in a coherent state $w_n(0) = \frac{\exp(-\frac{|\alpha|^2}{2})\alpha^n}{\sqrt{n!}}$.

In this case, by substituting (27) and (28) into (33) we have

$$
\begin{aligned}
P(t) = \quad & 2|\wp_{eg}|\int d^3p|\phi(\vec{p})|^2\sum_{n=0}^{\infty}Re\{[|C(1)|^2H(A_n,B_t) \\
& H^*(A_n+1,B_t)+C(1)C^*(2)H(A_n,B_t)_1F_1^*(\frac{-1}{2}(A_n+1),\frac{1}{2};B_t^2) \\
+ \quad & C(2)C^*(1)H^*(A_n+1,B_t)_1F_1(-A_n,\frac{1}{2};B_t^2) \\
+ \quad & |C(2)|_1^2F_1(-A_n),\frac{1}{2};B_t^2)_1F_1^*(\frac{-1}{2}(A_n+1),\frac{1}{2};B_t^2)]\exp(it\Delta_1(\vec{p},\vec{g},t))\},
\end{aligned} \tag{35}
$$

where $C_{\vec{p}_0}(1)=\frac{C_{1,\vec{p}_0}}{C_{\vec{p}_0}}$, $C_{\vec{p}_0}(2)=\frac{C_{2,\vec{p}_0}}{C_{\vec{p}_0}}$ with

$$
\begin{aligned}
C_{1,\vec{p}_0} = \quad & \frac{\exp(-\frac{|\alpha|^2}{2})\alpha^n}{\sqrt{2(n!)}}({}_1F_1(\frac{-1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};D^2(\vec{p}_0)) \\
- \quad & (\frac{\alpha}{\sqrt{n+1}})_1F_1(-A_n(\vec{p}_0),\frac{1}{2};D^2(\vec{p}_0))),
\end{aligned} \tag{36}
$$

and

$$
\begin{aligned}
C_{2,\vec{p}_0} = \quad & \frac{\exp(-\frac{|\alpha|^2}{2})\alpha^n}{\sqrt{2(n!)}}(H(A_n(\vec{p}_0)+1,-D(\vec{p}_0)) \\
- \quad & (\frac{\alpha}{\sqrt{n+1}})H(A_n(\vec{p}_0),-D(\vec{p}_0))),
\end{aligned} \tag{37}
$$

so that

$$
\begin{aligned}
C_{\vec{P}_0} = \quad & H(A_n(\vec{p}_0),-D(\vec{p}_0))_1F_1(-\frac{1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};D^2(\vec{p}_0)) \\
- \quad & H(A_n(\vec{p}_0)+1,-D(\vec{p}_0))_1F_1(-A_n(\vec{p}_0),\frac{1}{2};D^2(\vec{p}_0)),
\end{aligned} \tag{38}
$$

and we have $A_n(\vec{p}_0)=-(2+i\beta_{\vec{p}_0})$ $\beta_{\vec{p}_0}=\frac{\Omega_n(\vec{p}_0,\vec{g})-\Delta_0(\vec{p}_0)^2}{2\vec{q}.\vec{g}}$, $B_t(\vec{p}_0)=(\gamma t-\eta_{\vec{p}_0})(1+i)$, $\gamma=\frac{\sqrt{2}}{2}\vec{q}.\vec{g}$, $\eta_{\vec{p}_0}=\frac{\sqrt{2}\Delta_0(\vec{p}_0)}{4\sqrt{\vec{q}.\vec{g}}}$, $D_{\vec{p}_0}=\eta_{\vec{p}_0}(1+i)$. Figure 1a show the dipole moment evolution assuming $q=10^7m^{-1}$, $p_0=10^{-26}\frac{Kg.m}{s}$,$g=9.8\frac{m}{s^2}$,$\omega_{rec}=.5\times10^6\frac{rad}{s}$,$\lambda=9.7\times10^6\frac{rad}{s}$,$\triangle_0=8.5\times10^7\frac{rad}{s}$ and $\varphi=0$ [30-33]. Here we consider a two-level atom in a coherent superposition of the excited state and the ground state traversing in horizontal direction with the momentum vector $\vec{p_0}$ of an optical cavity in the presence of gravitatonal field so that $\vec{p_0}.\vec{g}=0$ and $\vec{p_0}.\vec{q}=p_0q\cos\theta,\vec{q}.\vec{g}=qg\sin\theta$ where $\theta$ is the angle between $\vec{q}$ and $\vec{p_0}$, and $\frac{\pi}{2}-\theta$ is the angle between $\vec{q}$ and $\vec{g}$. Before a given

atom passes through the cavity, the cavity mode is prepared in the coherent state. In figure 1b we consider small gravitational influence.This means very small $\vec{q}.\vec{g}$, i.e., the momentum transfer from the laser beam to the atom is only slightly altered by the gravitational acceleration because the latter is very small or nearly perpendicular to the laser beam. With comparing figures 1a and 1b we can see gravitational influence on the dipole moment by appearing oscillations such as collapses and revivals.

### 4b. Atomic Inversion

Another important quantity is the atomic population inversion [34] which is given by the expression

$$w(\vec{p},\vec{g},t) = \langle\psi(t)|\sigma_z|\psi(t)\rangle, \tag{39}$$

where from (18) we obtain

$$w(\vec{p},\vec{g},t) = \sum_{n=0} \int d^3p[|\psi_1|^2 - |\psi_2|^2]. \tag{40}$$

Therefore, by substituting from (27) and (28) into (40) and with assumes which we have used in the sub-section 4a, we can obtain

$$\begin{aligned} w(\vec{p}_0,\vec{g},t) = \quad & \sum_{n=0}^{\infty}\{|C_{1,\vec{p}_0}|^2[|H(A_n(\vec{p}_0),B_t(\vec{p}_0))|^2 \\ - \quad & |H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))|^2] + |C_{2,\vec{p}_0}|^2[|{}_1F_1(-A_n(\vec{p}_0),\frac{1}{2};B_t^2(\vec{p}_0))|^2 \\ - \quad & |{}_1F_1(-\frac{1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};B_t^2(\vec{p}_0))|^2] + 2Re[C_{1,\vec{p}_0}C^*_{2,\vec{p}_0} \\ & (H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))_1F_1^*(-A_n(\vec{p}_0),\frac{1}{2};B_t^2(\vec{p}_0)) \\ - \quad & H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))_1F_1^*(-\frac{1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};B_t^2(\vec{p}_0)))]\}, \end{aligned} \tag{41}$$

where we have defined all functions in terms of $\vec{p}_0$ in the sub-section 4a. Figures 2a and 2b have plotted with the same corresponding data, respectively, used in figures 1a and 1b. The gravitational field affect in the inversion population by appearing collapse and revival times so that we can see in figure 2a. In figure 2b we consider $\vec{q}.\vec{g} = 0$ so that we can not see collapse and revival times as well as figure 2a.

On the other hand, we calculate the collapse and revival times [35-38].

We show that these times depend on the gravitational field. An estimate of $t_c$ and $t_r$ can be therefore be obtained from the conditions

$$(\Omega_{\langle n\rangle+\sqrt{\langle n\rangle}} - \Omega_{\langle n\rangle-\sqrt{\langle n\rangle}})t_c \sim 1, \tag{42}$$

and

$$(\Omega_{\langle n\rangle} - \Omega_{\langle n\rangle-1})t_r \sim 2m\pi(m = 1, 2, 3, ...), \tag{43}$$

with rabi frequency

$$\Omega_n = (c_n^2 + d^2)^{\frac{1}{4}} \exp(\frac{i\varphi_n}{2}), \tag{44}$$

where

$$\tan(\varphi_n) = \frac{d}{c_n}, c_n = \triangle_0^2 + \lambda^2(n+1), d = 2\vec{q}.\vec{g}. \tag{45}$$

Therefore,we obtain real part of the collapse and revival times

$$t_c = \frac{r_{1c}\cos(\varphi_{1c}) - r_{2c}\cos(\varphi_{2c})}{(r_{1c}\cos(\varphi_{1c}) - r_{2c}\cos(\varphi_{2c}))^2 + (r_{1c}\sin(\varphi_{1c}) - r_{2c}\sin(\varphi_{2c}))^2}, \tag{46}$$

and

$$t_r = \frac{2m\pi r_{1r}\cos(\varphi_{1r}) - r_{2r}\cos(\varphi_{2r})}{(r_{1r}\cos(\varphi_{1r}) - r_{2r}\cos(\varphi_{2r}))^2 + (r_{1r}\sin(\varphi_{1r}) - r_{2r}\sin(\varphi_{2r}))^2}, \tag{47}$$

where

$$r_{1c} = (c^2_{\langle n\rangle+\sqrt{\langle n\rangle}} + d^2)^{\frac{1}{4}}, r_{2c} = (c^2_{\langle n\rangle-\sqrt{\langle n\rangle}} + d^2)^{\frac{1}{4}}, \tag{48}$$

and

$$r_{1r} = (c^2_{\langle n\rangle} + d^2)^{\frac{1}{4}}, r_{2r} = (c^2_{\langle n\rangle-1} + d^2)^{\frac{1}{4}}, \tag{49}$$

with

$$\cos(\varphi_{1c}) = (1 + \frac{d^2}{c^2_{\langle n\rangle+\sqrt{\langle n\rangle}}})^{-\frac{1}{2}}, \cos(\varphi_{2c}) = (1 + \frac{d^2}{c^2_{\langle n\rangle-\sqrt{\langle n\rangle}}})^{-\frac{1}{2}}, \tag{50}$$

and

$$\cos(\varphi_{1r}) = (1 + \frac{d^2}{c^2_{\langle n\rangle}})^{-\frac{1}{2}}, \cos(\varphi_{2r}) = (1 + \frac{d^2}{c^2_{\langle n\rangle-1}})^{-\frac{1}{2}}. \tag{51}$$

From (), () and () we can see that gravitational field affect the collapse and revival times. Moreover, we obtain the collapse and revival times by $\lambda t_c = 4.3$ and $\lambda t_r = 1.9$, respectively, with the same corresponding data used in the sub-section 4a. Therefore, the collapse and revival times that we have obtained from () and () are the same with the collapse and revival

times that we have shown in figure 2a.

### 4c. Atomic momentum diffusion

The next quantity is the atomic momentum diffusion which is given by

$$\Delta p = (\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2)^{\frac{1}{2}}. \tag{52}$$

By using (18) and $\hat{p}|p\rangle = p|p\rangle$, we obtain

$$\Delta p = \{[\sum_{n=0}^{\infty} \int d^3 p p^2 (|\psi_1|^2 + |\psi_2|^2)] - [\sum_{n=0}^{\infty} \int d^3 p p (|\psi_1|^2 + |\psi_2|^2)]^2\}^{\frac{1}{2}}. \tag{53}$$

Now we substitute (27) and (28) into (53) with assumes which is used in the sub-section 4a so that we can obtain

$$\begin{aligned}
\Delta p = \quad & \{[\sum_{n=0}^{\infty} p_0^2 (|C_{1,\vec{p}_0}|^2 [|H(A_n(\vec{p}_0), B_t(\vec{p}_0))|^2 \qquad (54)\\
+ \quad & |H(A_n(\vec{p}_0) + 1, B_t(\vec{p}_0))|^2] + |C_{2,\vec{p}_0}|^2 [|_1F_1(-A_n(\vec{p}_0), \frac{1}{2}; B_t^2(\vec{p}_0))|^2 \\
+ \quad & |_1F_1(-\frac{1}{2}(A_n(\vec{p}_0) + 1), \frac{1}{2}; B_t^2(\vec{p}_0))|^2] + 2Re[C_{1,\vec{p}_0} C_{2,\vec{p}_0}^* \\
& (H(A_n(\vec{p}_0) + 1, B_t(\vec{p}_0))_1F_1^*(-A_n(\vec{p}_0), \frac{1}{2}; B_t^2(\vec{p}_0)) \\
+ \quad & H(A_n(\vec{p}_0) + 1, B_t(\vec{p}_0))_1F_1^*(-\frac{1}{2}(A_n(\vec{p}_0) + 1), \frac{1}{2}; B_t^2(\vec{p}_0)))])] \\
- \quad & [\sum_{n=0}^{\infty} p_0 (|C_{1,\vec{p}_0}|^2 [|H(A_n(\vec{p}_0), B_t(\vec{p}_0))|^2 \\
+ \quad & |H(A_n(\vec{p}_0) + 1, B_t(\vec{p}_0))|^2] + |C_{2,\vec{p}_0}|^2 [|_1F_1(-A_n(\vec{p}_0), \frac{1}{2}; B_t^2(\vec{p}_0))|^2 \\
+ \quad & |_1F_1(-\frac{1}{2}(A_n(\vec{p}_0) + 1), \frac{1}{2}; B_t^2(\vec{p}_0))|^2] + 2Re[C_{1,\vec{p}_0} C_{2,\vec{p}_0}^* \\
& (H(A_n(\vec{p}_0) + 1, B_t(\vec{p}_0))_1F_1^*(-A_n(\vec{p}_0), \frac{1}{2}; B_t^2(\vec{p}_0)) \\
+ \quad & H(A_n(\vec{p}_0) + 1, B_t(\vec{p}_0))_1F_1^*(-\frac{1}{2}(A_n(\vec{p}_0) + 1), \frac{1}{2}; B_t^2(\vec{p}_0)))])]^2\}^{\frac{1}{2}},
\end{aligned}$$

where we apply all assumes in the sub-section 4a.

### 4d. The Probability distribution of the cavity-field

The probability distribution function $p(n)$ that there are n photons in the cavity-field is given by

$$P(n) = |\langle n|\psi(t)\rangle|^2. \quad (55)$$

By using the expressions (27) and (28) which represent the probability amplitudes, we have

$$P(n) = \int d^3p[|\psi_1|^2 + |\psi_2|^2]. \quad (56)$$

Therefore, by assumes which is used in the sub-section 4a, we obtain the probability distribution function $p(n)$ at time $t = \tau$

$$\begin{aligned} p(n) = \quad & \{|C_{1,\vec{p}_0}|^2[|H(A_n(\vec{p}_0), B_\tau(\vec{p}_0))|^2 \quad (57)\\ + \quad & |H(A_n(\vec{p}_0)+1, B_\tau(\vec{p}_0))|^2] + |C_{2,\vec{p}_0}|^2[|{}_1F_1(-A_n(\vec{p}_0), \frac{1}{2}; B_\tau^2(\vec{p}_0))|^2 \\ + \quad & |{}_1F_1(-\frac{1}{2}(A_n(\vec{p}_0)+1), \frac{1}{2}; B_\tau^2(\vec{p}_0))|^2] + 2Re[C_{1,\vec{p}_0}C^*_{2,\vec{p}_0} \\ & (H(A_n(\vec{p}_0)+1, B_\tau(\vec{p}_0))_1F_1^*(-A_n(\vec{p}_0), \frac{1}{2}; B_\tau^2(\vec{p}_0)) \\ + \quad & H(A_n(\vec{p}_0)+1, B_\tau(\vec{p}_0))_1F_1^*(-\frac{1}{2}(A_n(\vec{p}_0)+1), \frac{1}{2}; B_\tau^2(\vec{p}_0)))]\}, \end{aligned}$$

where we have introduced the functions in terms of $\vec{p}_0$ in the sub-section 4a. Moreover, in figures 3a and 3b we consider the same corresponding data, respectively, used in figures 1a and 1b with $\alpha = 2$,$\tau = 1.4 \times 10^{-6} sec.$ With comparing figures 3a and 3b we may see that the gravitational field affect in the probability distribution of the cavity-field.

### 4e. Photon Counting Statistics

We investigate another parameter for field so-called Mandel parameter $Q(t)$ [39,40]

$$Q(t) = \frac{(\langle n(t)^2\rangle - \langle n(t)\rangle^2)}{\langle n(t)\rangle} - 1, \quad (58)$$

where $Q = 0$, $Q < 0$ and $Q > 0$ for field show Poissonian, sub-Poissonian and super-Poissonian statistic, respectively. We define $\langle n(t)\rangle = \sum_{n=0}^{\infty} n(t)P(n)$ and we have

$$Q(t) = (\{[\sum_{n=0}^{\infty} n^2(t)P(n)] - [\sum_{n=0}^{\infty} n(t)P(n)]^2\}[\sum_{n=0}^{\infty} n(t)P(n)]^{-1}) - 1. \quad (59)$$

Therefore, by the assumes which are used in sub-sections 4a and 4d we obtain

$$
\begin{aligned}
Q = \quad & (\{[\sum_{n=0}^{\infty} n^2\{|C_{1,\vec{p}_0}|^2[|H(A_n(\vec{p}_0),B_t(\vec{p}_0))|^2 \qquad (60)\\
+ \quad & |H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))|^2] + |C_{2,\vec{p}_0}|^2[|{}_1F_1(-A_n(\vec{p}_0),\frac{1}{2};B_t^2(\vec{p}_0))|^2\\
+ \quad & |{}_1F_1(-\frac{1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};B_t^2(\vec{p}_0))|^2] + 2Re[C_{1,\vec{p}_0}C^*_{2,\vec{p}_0}\\
& (H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))_1F_1^*(-A_n(\vec{p}_0),\frac{1}{2};B_t^2(\vec{p}_0))\\
+ \quad & H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))_1F_1^*(-\frac{1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};B_t^2(\vec{p}_0)))]\}]\\
- \quad & [\sum_{n=0}^{\infty}\{|C_{1,\vec{p}_0}|^2[|H(A_n(\vec{p}_0),B_t(\vec{p}_0))|^2\\
+ \quad & |H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))|^2] + |C_{2,\vec{p}_0}|^2[|{}_1F_1(-A_n(\vec{p}_0),\frac{1}{2};B_t^2(\vec{p}_0))|^2\\
+ \quad & |{}_1F_1(-\frac{1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};B_t^2(\vec{p}_0))|^2] + 2Re[C_{1,\vec{p}_0}C^*_{2,\vec{p}_0}\\
& (H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))_1F_1^*(-A_n(\vec{p}_0),\frac{1}{2};B_t^2(\vec{p}_0))\\
+ \quad & H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))_1F_1^*(-\frac{1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};B_t^2(\vec{p}_0)))]\}]^2\}\\
& [\sum_{n=0}^{\infty}\{|C_{1,\vec{p}_0}|^2[|H(A_n(\vec{p}_0),B_t(\vec{p}_0))|^2\\
+ \quad & |H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))|^2] + |C_{2,\vec{p}_0}|^2[|{}_1F_1(-A_n(\vec{p}_0),\frac{1}{2};B_t^2(\vec{p}_0))|^2\\
+ \quad & |{}_1F_1(-\frac{1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};B_t^2(\vec{p}_0))|^2] + 2Re[C_{1,\vec{p}_0}C^*_{2,\vec{p}_0}\\
& (H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))_1F_1^*(-A_n(\vec{p}_0),\frac{1}{2};B_t^2(\vec{p}_0))\\
+ \quad & H(A_n(\vec{p}_0)+1,B_t(\vec{p}_0))_1F_1^*(-\frac{1}{2}(A_n(\vec{p}_0)+1),\frac{1}{2};B_t^2(\vec{p}_0)))]\}]^{-1}) - 1,
\end{aligned}
$$

where we have defined the functions in terms of $\vec{p}_0$ in the sub-section 4a. In figure 4a with an initial coherent state for field and the same data used in figure 1a we can see the mandel parameter until second order in presence of the gravitational field is negative. Therefore, the statistics is sub-Poissonian. Moreovere, in $\lambda t > 0.7$, the mandel parameter increases and in $\lambda t = 0.77$, this parameter is minimum. In figure 4b the mandel parameter in $\lambda t > 0.7$

decreases when $\vec{q}.\vec{g}$ is very small.

### 4f. Quadrature Squeezing of the Cavity-Field

Now we investigate the quadrature squeezing of the radiation field [41-45] in the presence of gravitational field. We introduce the Hermitian amplitude operators

$$\hat{X}_1 = \frac{1}{2}(\hat{a} + \hat{a}^{\dagger}), \hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^{\dagger}), \tag{61}$$

where $\hat{a}$ and $\hat{a}^{\dagger}$ obey the communication relation $[\hat{a}, \hat{a}^{\dagger}] = 1$. A squeezed state of the radiation field is obtained if

$$\langle(\triangle\hat{X}_i)^2\rangle < \frac{1}{4}, (i = 1 or 2), \tag{62}$$

where

$$\langle(\triangle\hat{X}_i)^2\rangle = \langle\hat{X}_i^2\rangle - \langle\hat{X}_i\rangle^2. \tag{63}$$

The degree of squeezing can be measured by the squeezing parameter $S_i, (i = 1 or 2)$ defined by

$$S_i = 4\langle(\triangle\hat{X}_i)^2\rangle - 1. \tag{64}$$

Therefore, from (18),(27) and (28) we obtain the squeezing parameter $S_i, (i = 1 or 2)$

$$S_1 = (\langle\hat{a}^2\rangle - \langle\hat{a}\rangle^2) + (\langle\hat{a}^{\dagger 2}\rangle - \langle\hat{a^{\dagger}}\rangle^2) + 2(\langle\hat{a}^{\dagger}\hat{a}\rangle - \langle\hat{a}^{\dagger}\rangle\langle\hat{a}\rangle), \tag{65}$$

and

$$S_2 = -(\langle\hat{a}^2\rangle - \langle\hat{a}\rangle^2) - (\langle\hat{a}^{\dagger 2}\rangle - \langle\hat{a^{\dagger}}\rangle^2) + 2(\langle\hat{a}^{\dagger}\hat{a}\rangle - \langle\hat{a}^{\dagger}\rangle\langle\hat{a}\rangle), \tag{66}$$

where

$$\langle\hat{a}\rangle = \int d^3p \sum_{n=0}^{\infty}(\sqrt{n}\psi_{1n}\psi^*_{1(n-1)} + \sqrt{n+1}\psi_{2n}\psi^*_{2(n-1)}), \tag{67}$$

$$\langle\hat{a^{\dagger}}\rangle = \int d^3p \sum_{n=0}^{\infty}(\sqrt{n+1}\psi_{1n}\psi^*_{1(n+1)} + \sqrt{n+2}\psi_{2n}\psi^*_{2(n+1)}), \tag{68}$$

and

$$\langle\hat{a}^2\rangle = \int d^3p \sum_{n=0}^{\infty}(\sqrt{n(n-1)}\psi_{1n}\psi^*_{1(n-2)} + \sqrt{n(n+1)}\psi_{2n}\psi^*_{2(n-2)}), \tag{69}$$

$$\langle \hat{a}^{\dagger 2} \rangle = \int d^3p \sum_{n=0}^{\infty} (\sqrt{(n+1)(n+2)}\psi_{1n}\psi^*_{1(n+2)} + \sqrt{(n+2)(n+3)}\psi_{2n}\psi^*_{2(n+3)}), \tag{70}$$

with

$$\langle \hat{a}^{\dagger} \hat{a} \rangle = \int d^3p \sum_{n=0}^{\infty} (n\psi_{1n}\psi^*_{1n} + (n+1)\psi_{2n}\psi^*_{2(n-1)}), \tag{71}$$

where we define from (27) and (28) $\psi_{1n} = \psi_1(t)$ and $\psi_{2n} = \psi_2(t)$, respectively, and from sub-section 4a we apply all assumes and the initial conditions. In figures 5a and 5b we have plotted the squeezing parameters $S_i, (i = 1 or 2)$ versus the scaled time $\lambda t$ for the same corresponding data, respectively, used in figures 1a and 1b. As it is seen, each of the two quadrature components exhibits squeezing in the course of time evolution. Because of the influence of gravitational field, each of the two quadrature components show oscillatory behavior.

## 5 Summary and conclusions

We have studied the temporal evolution of quantum statistical properties of an interacting atom-field system in the presence of a homogeneous gravitational field within the framework of the Jaynes-Cummings model. For this purpose, taking into account both the atomic motion and gravitational field a full quantum treatment of the internal and external dynamics of the atom has presented based on an alternative su(2) dynamical algebraic structure. By solving analytically the Schrödinger equation in the interaction picture, the evolving state of the system has found by which the influence of the gravitational field on the dynamical behavior of the atom-field system has explored. Assuming that initially the field has prepared in a coherent state and the two-level atom has prepared in a coherent superposition of the excited and ground states, the influence of gravity on the atomic dipole moment, collapses and revivals of the atomic motion, atomic momentum diffusion, photon counting statistics and quadrature squeezing of the radiation field has studied.

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**FIGURE CAPTIONS:**

**FIG. 1-4** The dipole moment evolution versus the scaled time $\lambda t$. Here $q = 10^7 m^{-1}$, $p_0 = 10^{-26}\frac{Kg.m}{s}$,$g = 9.8\frac{m}{s^2}$,$\omega_{rec} = .5 \times 10^6\frac{rad}{s}$, $\lambda = 9.7 \times 10^6\frac{rad}{s}$,$\triangle_0 = 8.5 \times 10^7\frac{rad}{s}$, $\varphi = 0$ and $c_e = c_g = \frac{1}{\sqrt{2}}$ with coherent state for initial cavity-field;

**a)**In the presence of gravitational field.
**b)**For $\vec{q}.\vec{g} = 0$.

**FIG. 5** The squeezing parameters versus the scaled time $\lambda t$ with the same corresponding data used in fig.1-4;

**a)** The squeezing parameter $S_1$ in the presence of gravitational field.

**b)** The squeezing parameter $S_2$ in the presence of gravitational field.